DISCOVERY
with Neville de Mestre

Paper sizes and mathematics

Reams of paper come in a standardised system of related sheet sizes. Most of us are familiar with the international paper sizes A4, A3 and B4, but there are others.

The ratio of the sides of any sheet in the series is such that if the paper is cut or folded in half on itself then the ratio of the sides remains unchanged.

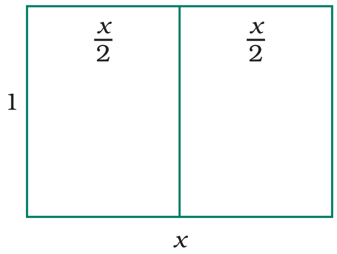

Figure 1

From Figure 1 it is seen that the full sheet and half sheet must be related by

$$\frac{x}{1} = \frac{1}{\left(\frac{x}{2}\right)}$$

Thus

$$x^2 = 2$$
$$x = \sqrt{2}$$
$$\approx 1.414$$

Due to this property of constant proportions when folding, artwork will enlarge or reduce photographically to fit any international paper size.

The A series is for general printed matter including stationery and publications. The basic standard sheet is designated A0 and measures 841 mm × 1189 mm, which is 0.999949 m^2 or approximately one square metre in area. Each number after the A indicates a halving of the preceding larger area. Thus A1 is half of A0, A2 is a quarter of A0, A3 is one-eighth of A0, etc. Therefore an A4 sheet is about $\frac{1}{16}$ m^2. Sheet sizes larger than A0 retain the A0 designation, but in addition have a numeral prefixed to indicate how many times larger than A0 they are. Thus 2A0 is approximately 2 m^2 and 4A0 is approximately 4 m^2. Table 1 gives the dimensions for some international paper sizes. Ask your students to determine the area of each size.

Table 1

Size	Dimensions (mm)
4A0	1682 × 2378
2A0	1189 × 1682
A0	841 × 1189
A1	594 × 841
A2	420 × 594
A3	297 × 420
A4	210 × 297
A5	148 × 210

A neat geometrical method of seeing various successive sizes on the same diagram is shown in Figure 2. Construct a 1:√2 rectangle and consider it as representing the A0 size. Now draw one diagonal. This diagonal contains the corner point of all smaller A sizes.

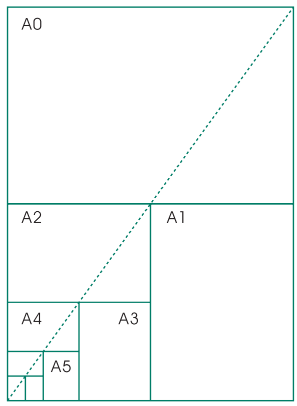

Figure 2

The B series is about halfway between successive A sizes. It is intended as an alternative to A sizes, and is used primarily for books, posters and wall charts. The average of the two dimensions of A0 is (841+1189)/2 = 1015, and so one basic dimension of the B series is taken as 1000 and the other is √2 times this value, or 1414 approximately. Ask your students to prepare a table containing the B-series dimensions from B0 to B5.

Let us now select one A4 sheet, and perform some mathematical magic with it

in a topological sense. I will show you how to cut a hole in this sheet so that an average-sized adult can easily pass through the hole without ripping the paper. First of all, we should recall the dimensions of this sheet. From Table 1 they are seen to be 210 mm × 297 mm = 0.06237 m^2. Suppose that I aim to cut a hole in the paper somehow, so that the continuous strip around the edge of the hole is 20 mm wide approximately. If I do not have to throw any paper away when I make my cut, then the circumference of the strip will be (210 × 297)/20 = 3118.5 mm. That is, the paper ring would be over 300 cm in circumference, which is certainly much larger than the average (non-obese) adult's girth. So all that remains after this little bit of basic mathematics is to figure out how to cut the paper so that none is lost, but it forms a paper ring 2 cm wide approximately.

To do this bit of mathematical magic you should fold the A4 sheet to A5 size, and then make six cuts of the paper through the folded edge as shown in Figure 3 by the solid lines, stopping about 20mm from the paper's edge. The cuts should be made so that the two outside sections are 17.5 mm wide and the five inner sections are 35 mm wide.

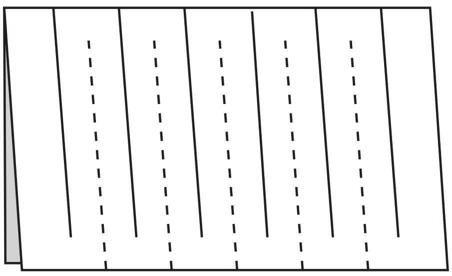

Figure 3

Next make five cuts of the inner sections on both sides of the folded paper as shown by the dashed lines, stopping about 10 mm from the fold line. Finally cut the five inner section fold lines that were 35 mm wide, but not the two outside fold lines. Your paper ring is now complete and can easily pass down over your head, shoulders and waist to the ground, where you can step out of it.

As a corollary, you can ask your students to make only five initial cuts through the fold line and to calculate where these cuts should be made, and what the circumference would now be for the resulting paper ring. Can they still fit through it? Repeat for four cuts initially.

Finally, ask your students to measure the size of a newspaper broadsheet. Take one of these broadsheets and fold it once on itself, producing two thicknesses of paper. When you do it again you have four thicknesses. How many thicknesses would you have if you folded it 20 times like this on itself? Your students should draw up a table as shown below and try to complete it up to 20 folds.

Table 2

FOLDS	THICKNESSES
1	2
2	4
3	8
4	16
5	32

It then becomes clear that after 20 folds the paper will have more than one million thicknesses. If we make the reasonable assumption that the broadsheet is 0.1 mm thick (10 thicknesses to 1mm), then the combined thickness of our 20 folds is over 100 metres. Wow!

Some discerning readers (or students) will tell you that a broadsheet cannot be physically folded 20 times. Try it and you will see that they are correct. However, instead of folding continually, we could cut along each fold line and stack. Again the height of the stack will be over 100 metres after 20 of these operations. You should ask your students to calculate the size of each member of the stack. Since the broadsheet was close to A2 size, it was about 250 000 mm^2 in area. Cutting this into one million equal pieces gives us 0.25 mm^2 for each piece. They would each be approximately 0.6 mm × 0.4 mm. They would be very tiny pieces of paper indeed, but if they could be stacked on top of each other without blowing away, they would reach up to a height of more than 100 metres.

Happy discoveries!